Student No.

Name: _____

Class: 12MTA21_____

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2021

YEAR 12

AP4

MATHEMATICS ADVANCED

Time allowed – 3 hours plus 10 minutes reading time

General Instructions	• Attempt all questions • Write your name and student number on the question paper • Write using black pen • NESA approved calculators may be used • The NESA reference sheet has been provided • For questions in Section II, show relevant mathematical reasoning and/or calculations
Total marks: 100	**Section I – 10 marks (pages 3 – 7)** • Attempt Questions 1-10 • Allow about 15 minutes for this section **Section II – 90 marks (pages 8 – 30)** • Attempt Questions 11-34 • Allow about 2 hours and 45 minutes for this section

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Section I
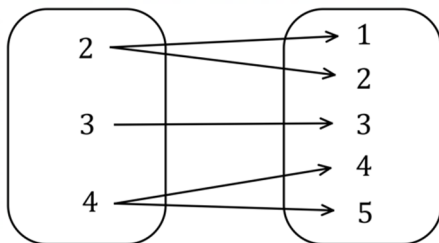
Use the multiple-choice answer sheet for Questions 1 – 10.

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1. If $f(x) = x^2 - 5x + 3$, what is $f(-3)$?

    (A) -2

    (B) -3
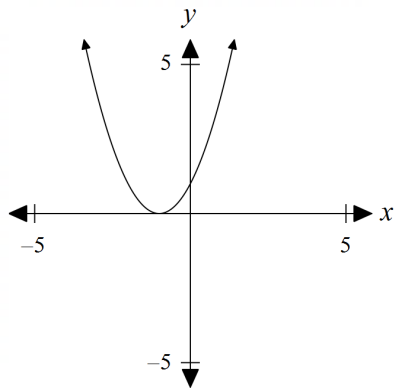
    (C) 9

    (D) 27

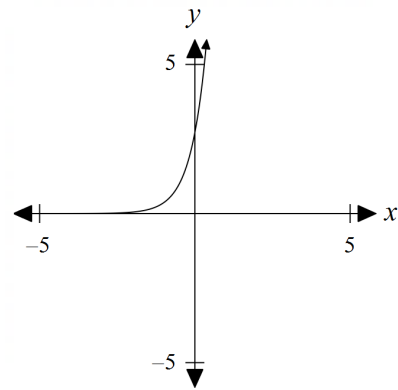2. What type of relation is shown?

    (A) Many-to-many

    (B) One-to-many

    (C) One-to-one

    (D) Many-to-one

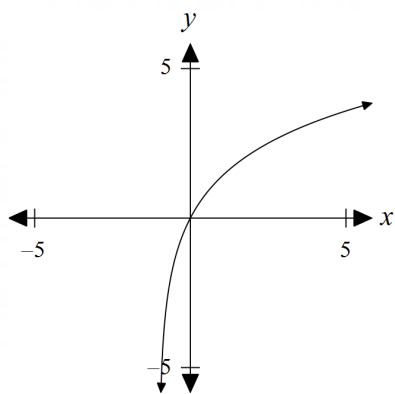**3.** Which of the following best represents the graph of $g(x) = e^{2\ln(x+1)}$?

(A)

(B)

(C)

(D)

4. The probability distribution of random variable X is shown below.

x	−3	−2	−1	0	1	2	3
P($X = x$)	0.05	0.05	a	0.20	0.15	a	0.05

What is the value of a?

(A)   0.15

(B)   0.20

(C)   0.25

(D)   0.30

5. The first term of an infinite geometric series is 12 and the limiting sum of that series is 15. What is the common ratio?

(A) $\dfrac{1}{5}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{1}{2}$

6. What is the amplitude and period for the function $f(x) = 4\sin\left(\dfrac{x+\pi}{3}\right)$?

(A) Amplitude 3 and period $\dfrac{\pi}{2}$

(B) Amplitude 3 and period 6π

(C) Amplitude 4 and period $\dfrac{\pi}{2}$

(D) Amplitude 4 and period 6π

7. Which interval gives the domain of the function $y = 2\sqrt{25 - x^2}$?

(A) $[-10, 10]$

(B) $[0, 10]$

(C) $[-5, 5]$

(D) $[0, 5]$

8. What is the area of the triangle given below?

48°

35 cm

NOT TO
SCALE

32°

49 cm
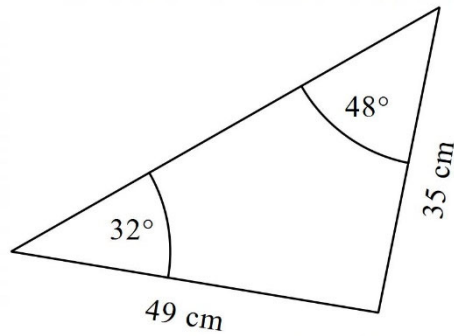
(A) $422 \ cm^2$

(B) $637 \ cm^2$

(C) $844 \ cm^2$

(D) $858 \ cm^2$

9. Which expression is the derivative of $\cos^2 3x$ when differentiated with respect to x ?

(A) $-6 \sin 3x \cos 3x$

(B) $-2 \sin 3x \cos 3x$

(C) $2 \sin 3x \cos 3x$

(D) $6 \sin 3x \cos 3x$

10. At which point on this curve are the first and second derivatives both negative?

(A)   *A*

(B)   *B*

(C)   *C*

(D)   *D*

END OF SECTION I

Section II

90 marks
Attempt Questions 11 – 34
Allow about 2 hours and 45 minutes for this section

Instructions
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)   **Marks**

A circle is given by the equation $x^2 + y^2 - 6x + 2y = 6$.   **2**
Find the centre and radius of the circle.

Question 12 (2 marks)

Differentiate: $y = \dfrac{e^{2x}}{x+1}$.   **2**

Question 13   (2 marks)

Find $\int (3x - 4)^8 \, dx$ **2**

Question 14   (2 marks)

Find $\int \dfrac{4 \sin \left(\frac{5x}{3}\right)}{7} \, dx$ **2**

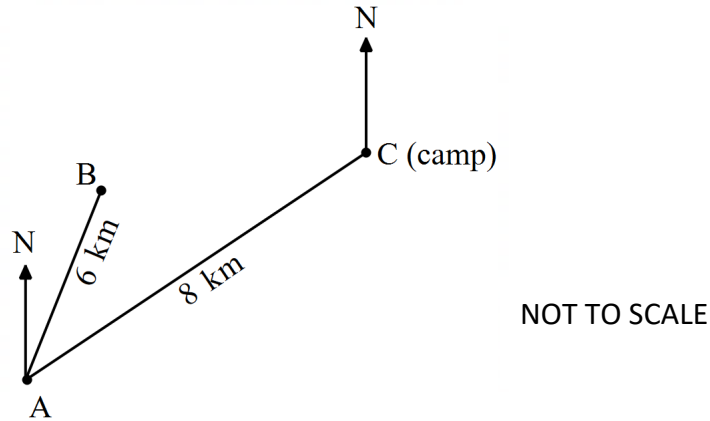---

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Question 15 on next page

Question 15   (3 marks)

A hiking group walked 8 km from their camp (C) on a bearing of 230° to point A.

They then walked 6 km on a bearing of 015° to point B.

The diagram below shows their path.

NOT TO SCALE

(a)   What is the size of $\angle CAB$?   **1**

(b)   Calculate the distance from B to C, correct to 1 decimal place.   **2**

Question 16   (4 marks)

The curve C has the equation $y = f(x)$, $x \neq 0$. The point $P(2,1)$ lies on C.

(a)   Find $f(x)$, given that $f'(x) = 3x^2 - 6 - \dfrac{8}{x^2}$.   **2**

(b)   What is the equation of the tangent to C at the point P?   **2**

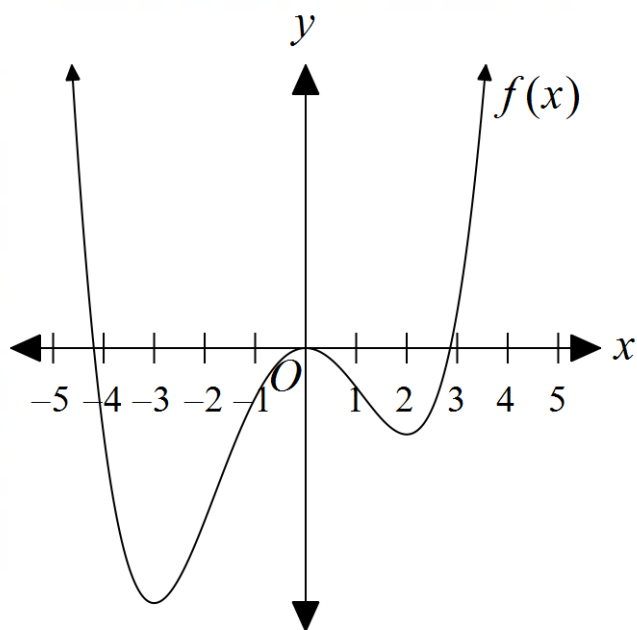---

Question 17 (2 marks)

The diagram below shows the graph of $y = f(x)$.

In the space below, sketch the derivative function, $y = f'(x)$. **2**

Question 18 on next page

Question 18   (2 marks)

Find $\displaystyle\int x^2 (x^3 + 5)^3 dx$ **2**

Question 19   (3 marks)

Consider the functions $f(x) = 2x^2 - 8x$ and $g(x) = x + 2$.

(a) Determine the composite function $f\big(g(x)\big)$.   **2**

(b) Using interval notation, state the range of the composite function.   **1**

Question 20 (2 marks) **Marks**

The graph below shows $y = f(x)$. **2**

On the same graph, sketch $y = 2 - f(x)$.
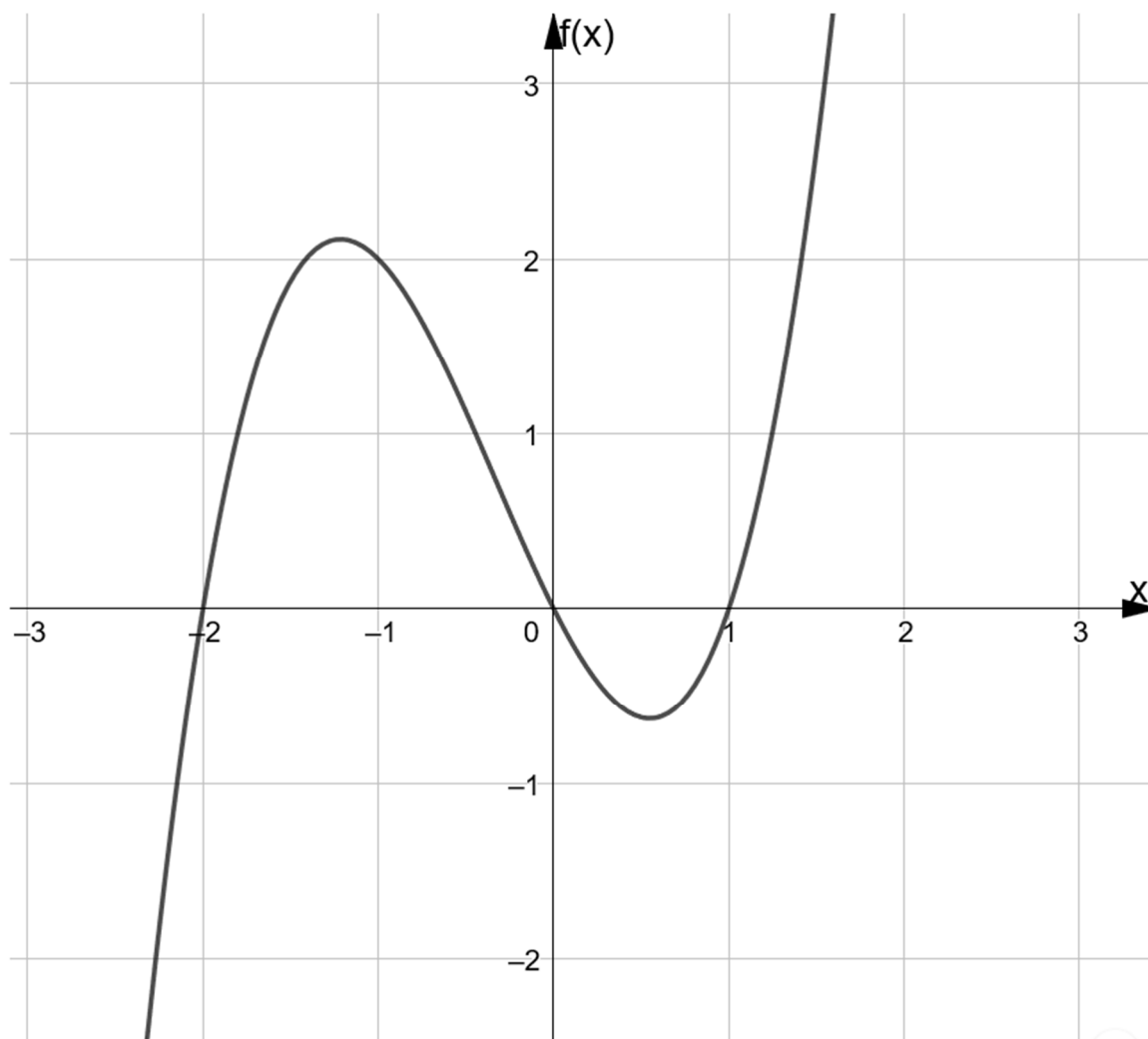
Question 21 on next page

Question 21   (2 marks)   **Marks**

Show that $\sin x + 1 + \cos x \cot x - \operatorname{cosec} x = 1$ **2**

Question 22   (2 marks)   **Marks**

Solve $2\cos(2x) = -\sqrt{3}$ for x where $0 \le x \le \pi$ **2**

Question 23 on next page

Question 23   (8 marks)

A function is given by $y = -x^3 + 9x^2 - 24x + 16$.

(a) Find any stationary points, determine their nature, and find any possible point/s of inflection.

Question 23 continued on next page

Question 23 continued

(b) Hence, or otherwise, sketch the curve labelling all important points.   **3**

Question 24   (3 marks)

(a) Complete the table of values for $y = \sqrt{1 - x^2}$.

Answer to 3 decimal places where required.

x	0	0.125	0.25	0.375	0.5
y			0.968		0.866

(b) Use the Trapezoidal rule with 4 subintervals, to estimate the value of

$$\int_0^{\frac{1}{2}} \sqrt{1 - x^2} \, dx$$

Question 25   (5 marks)

Lachlan works in a grocery store. He is making a stack of oranges against a sloping display panel.

The oranges are stacked in layers, as shown below. Each layer contains one orange less than the layer below it.

Top Layer

?

? nth Layer
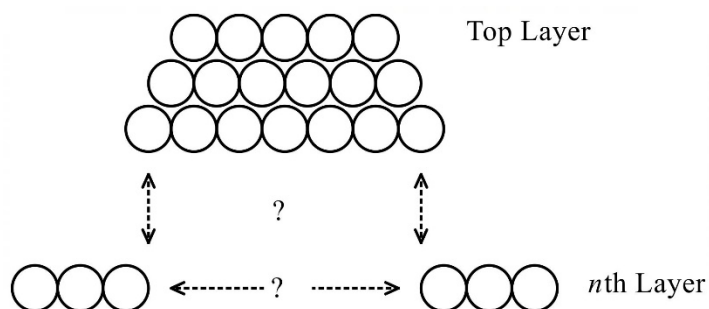
When he has finished, there are five oranges in the top layer, six in the next and so on.

There are n layers altogether.

Question 25 continued on next page

Question 25 continued

(a) Show that there are $\dfrac{1}{2}n(n+9)$ oranges in the stack. **2**

(b) If Lachlan has 300 oranges to create his display, how many full rows can he create, if the top row still contains five oranges? **3**

Question 26   (6 marks)

From a packet of mixed seeds it was estimated that the probability of any seed planted yielding a red rose was 0.02.

(a) Calculate the probability that from any two seeds planted there will be: **1**

   (i) two red roses

   (ii) at least one red rose   **2**

(b) How many seeds must be planted for you to be at least 98% certain of obtaining at least one red rose?   **3**

Question 27 on next page

Question 27   (3 marks)   **Marks**

Differentiate $f(x) = 2x^2 + 5x$ from first principles.   **3**

Question 28 on next page

Question 28   (6 marks)

Two curves $y = 2e^{-x}$ and $y = e^x - 1$ intersect at point P.

(a) Show that the coordinates of P are $(\ln 2 , 1)$.   **3**

Question 28 continues on next page

Question 28 continued

(b) Calculate the shaded area. **3**

Question 29 (3 marks) **Marks**

(a) Differentiate $\log_e(\cos x)$ with respect to x. **2**

(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \tan x \, dx$, correct to 2 decimal places. **2**

Question 30 (6 marks)

A spinner has the numbers 1 to 5 placed on it. The probability distribution for the spinner is shown below.

x	1	2	3	4	5
$P(X = x)$	0.1	0.25	0.4	0.1	0.15

(a) Find $P(2 < X \leq 5)$

1

(b) Find $P(X \leq 3 | X > 1)$

1

(c) Find the expected value of X.

2

(d) Determine the variance of X.

2

Question 31   (7 marks)

The bivariate data in the scatterplot below compares the recorded IQ Score (I) of students with their Mark on a Test (M) out of 100.
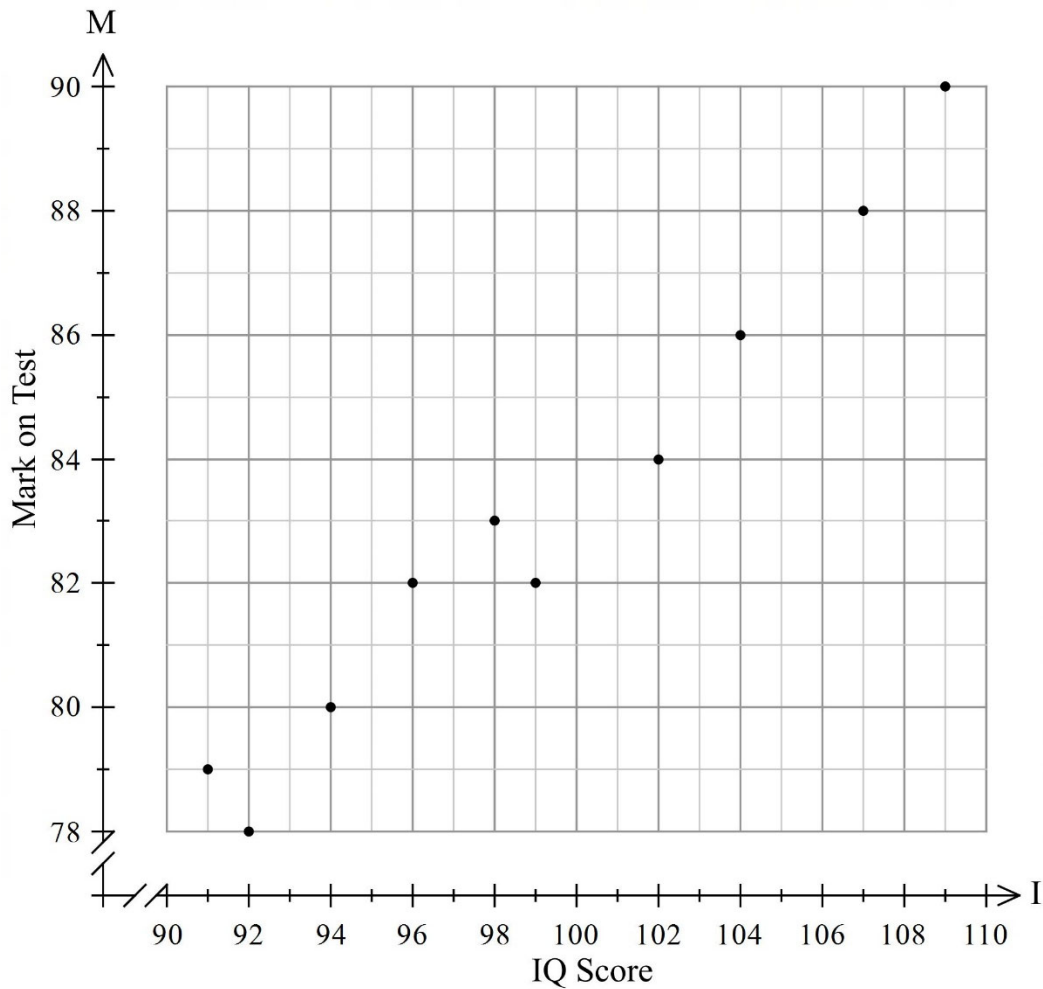
(a) Using your calculator, find (r) Pearson's correlation coefficient correct to 2   **3**
decimal places and describe the direction and strength of correlation this
data gives.

Question 31 continued on next page

(b) Using your calculator to find a line of best fit in the form $y = mx + c$.
Round both m and c to two decimal places.   **2**

(c) Use your equation to estimate the Mark for a student who recorded an IQ
score of 114.   **2**
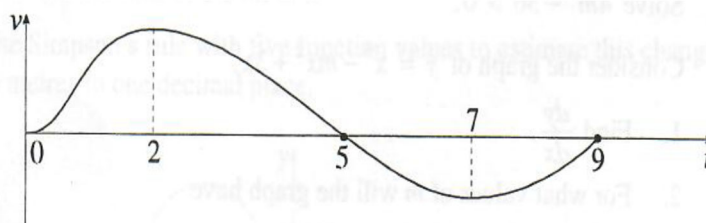
Is this a valid estimate? Justify your answer.

Question 32   (2 marks)   **Marks**

The graph below shows the velocity, v m/s, of a particle moving on a straight line,
for $0 \le t \le 9$.

State all times, or interval of time, for which:

(a) the particle is stationary   **1**

(b) the acceleration is negative.   **1**

Question 33   (5 marks)

The water level in an estuary is cyclical, with a maximum depth of 5 metres, a minimum depth of 2 metres, and the cycle repeats every 12 hours.

The last high tide was at 2:00 am.

The function of the form $D = k\cos\frac{\pi}{6}(t + b) + c$ models the water depth, where D is the water depth (in metres), t is the hours since 12:00 am (midnight) and k, b and c are constants.
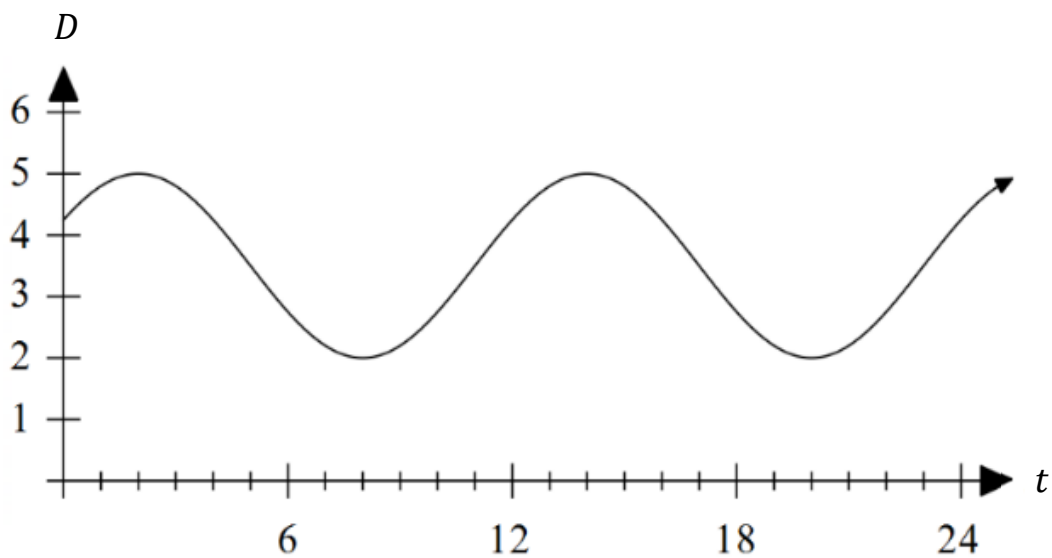
(a) Determine the values of k, b and c.   **3**

Question 33 continued on next page

Question 33 continued

(b) What is the rate of change of the water level at 12:00 pm? **2**

Question 34 on next page

Question 34   (7 marks)

The diagram above shows a rectangle $PQRS$ drawn in a sector OAB of a circle with a radius of 5 metres.

$\angle AOB = \theta$ and $\tan \theta = 2$, $OR = x$ m and $RS = y$ m.

 (a) Show that the perimeter, P m, of the rectangle is given by: **3**

$$P = 2x + 2\sqrt{25 - 4x^2}$$

(b) Find the maximum perimeter of the rectangle. **4**
     Leave your answer in exact form.

End of Paper

Section II   Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II   Extra writing space

If you use this space, clearly indicate which question you are answering.

Year 12 Mathematics Advanced Section I – Answer Sheet

Name: _____   **Class:** _____

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 =   (A) 2   (B) 6   (C) 8   (D) 9

A ◯   B ⬤   C ◯   D ◯

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⬤   B ⊗   C ◯   D ◯

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.   ↙ *correct*

A ⬤   B ⊗   C ◯   D ◯

1.   A ◯   B ◯   C ◯   D ◯

2.   A ◯   B ◯   C ◯   D ◯

3.   A ◯   B ◯   C ◯   D ◯

4.   A ◯   B ◯   C ◯   D ◯

5.   A ◯   B ◯   C ◯   D ◯

6.   A ◯   B ◯   C ◯   D ◯

7.   A ◯   B ◯   C ◯   D ◯

8.   A ◯   B ◯   C ◯   D ◯

9.   A ◯   B ◯   C ◯   D ◯

10.   A ◯   B ◯   C ◯   D ◯

Student No.

Name: _____SOLUTIONS_____

Class: 12MTA21_____

CHERRYBROOK TECHNOLOGY HIGH SCHOOL

2021

YEAR 12

AP4

MATHEMATICS ADVANCED

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Total marks: 100

Section I – 10 marks (pages 3 – 7)
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8 – 34)
- Attempt Questions 11-34
- Allow about 2 hours and 45 minutes for this section

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Section I

10 marks
Attempt Questions 1 – 10
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Use the multiple-choice answer sheet for Questions 1 – 10.

1. If $f(x) = x^2 - 5x + 3$, what is $f(-3)$?

   (A) -2

   (B) -3

   (C) 9

   (D) 27

$$f(-3) = (-3)^2 - 5(-3) + 3$$
$$= 27$$

2. What type of relation is shown?
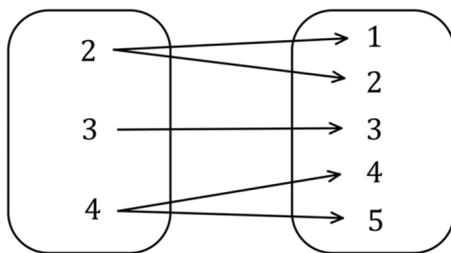
   (A) Many-to-many

   (B) One-to-many

   (C) One-to-one
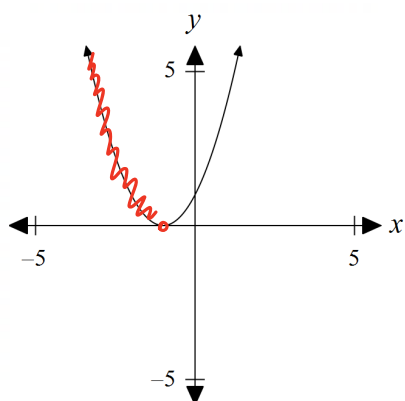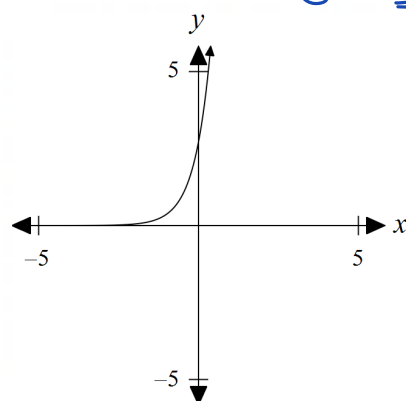
   (D) Many-to-one

3. Which of the following best represents the graph of $g(x) = e^{2\ln(x+1)}$?

$g(x) = e^{\ln(x+1)^2}$
$= (x+1)^2$
$x > -1$

(A)

(B)

(C)
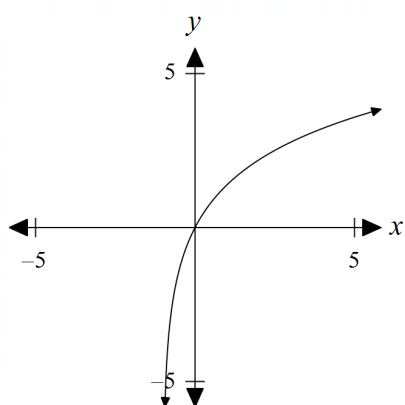
(D)
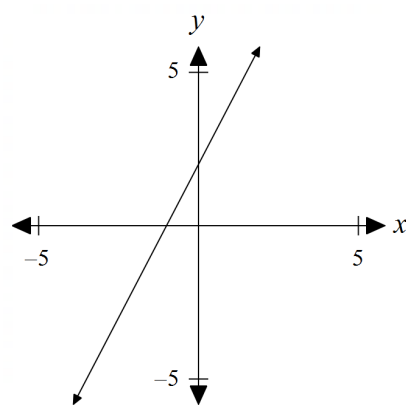
4. The probability distribution of random variable X is shown below.

x	−3	−2	−1	0	1	2	3
$P(X= x)$	0.05	0.05	a	0.20	0.15	a	0.05

What is the value of a?

(A)   0.15

(B)   0.20

(C)   0.25

(D)   0.30

5. The first term of an infinite geometric series is 12 and the limiting sum of that series is 15. What is the common ratio?

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

$a = 12$

$S = 15$

$15 = \frac{12}{1-r}$

$1 - r = \frac{12}{15}$

$r = \frac{3}{15} = \frac{1}{5}$

6. What is the amplitude and period for the function $f(x) = 4\sin\left(\frac{x+\pi}{3}\right)$?

(A) Amplitude 3 and period $\frac{\pi}{2}$

(B) Amplitude 3 and period 6π

(C) Amplitude 4 and period $\frac{\pi}{2}$

(D) Amplitude 4 and period 6π

Amplitude $= 4$

$T = \frac{2\pi}{\frac{1}{3}} = 6\pi$

7. Which interval gives the domain of the function $y = 2\sqrt{25 - x^2}$?

(A) $[-10, 10]$

(B) $[0, 10]$

(C) $[-5, 5]$

(D) $[0, 5]$

8. What is the area of the triangle given below?
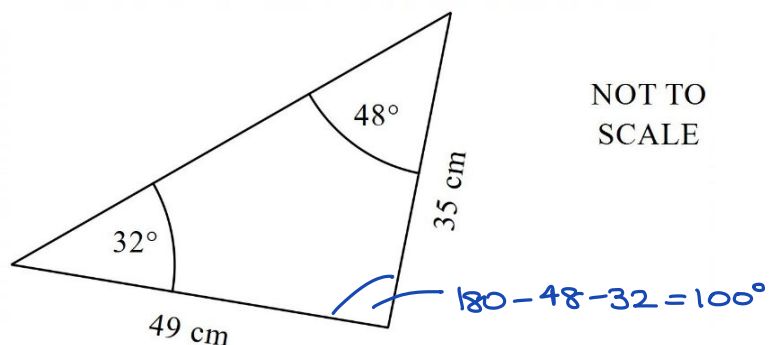
NOT TO SCALE

$180 - 48 - 32 = 100°$

$A = \frac{1}{2}(35)(49) \sin 100°$

$= 844$

(A) $422 \ cm^2$

(B) $637 \ cm^2$

(C) $844 \ cm^2$

(D) $858 \ cm^2$

9. Which expression is the derivative of $\cos^2 3x$ when differentiated with respect to x ?
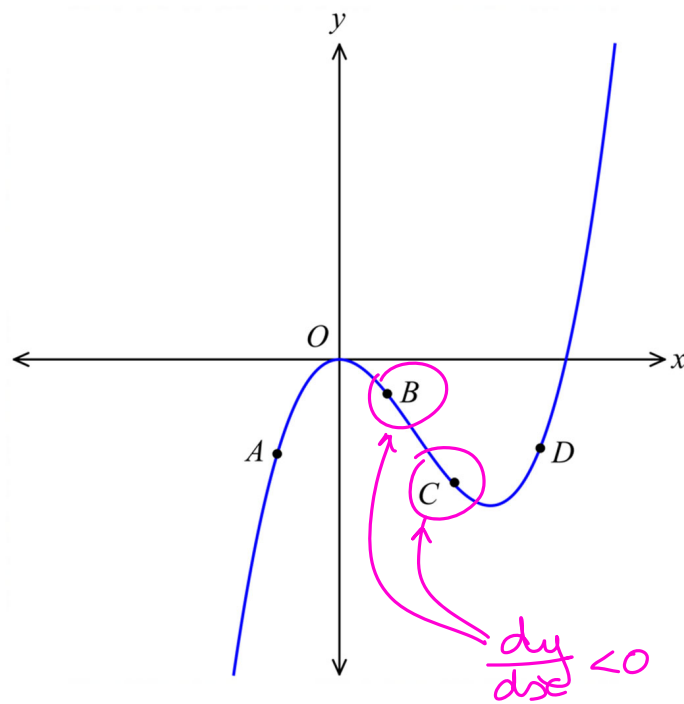
(A) $-6 \sin 3x \cos 3x$

(B) $-2 \sin 3x \cos 3x$

(C) $2 \sin 3x \cos 3x$

(D) $6 \sin 3x \cos 3x$

$\frac{d}{dx}(\cos 3x)^2 = 2\cos 3x \cdot -3\sin 3x$

$= -6\sin 3x \cos 3x$

10. At which point on this curve are the first and second derivatives both negative?

(A)   *A*

(B)   *B*

(C)   *C*

(D)   *D*

END OF SECTION I

Section II

90 marks
Attempt Questions 11 – 34
Allow about 2 hours and 45 minutes for this section

Instructions
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11   (2 marks)   **Marks**

A circle is given by the equation $x^2 + y^2 - 6x + 2y = 6$.   **2**

Find the centre and radius of the circle.

$$x^2 - 6x + 9 + y^2 + 2y + 1 = 6 + 9 + 1$$

$$(x-3)^2 + (y+1)^2 = 16$$

Centre $(3, -1)$ radius $= 4$

Question 12   (2 marks)

Differentiate: $y = \dfrac{e^{2x}}{x+1}$.   **2**

$$u = e^{2x} \qquad\qquad v = x+1$$

$$u' = 2e^{2x} \qquad\qquad v' = 1$$

$$\frac{dy}{dx} = \frac{(x+1) \cdot 2e^{2x} - e^{2x}(1)}{(x+1)^2}$$

$$= \frac{e^{2x}(2x+2-1)}{(x+1)^2}$$

$$= \frac{e^{2x}(2x+1)}{(x+1)^2}$$

Question 13   (2 marks)

Find $\int (3x - 4)^8 \, dx$ **2**

$$\frac{d}{dx} (3x-4)^9 = 9(3x-4)^8 \cdot 3$$

$$= 27(3x-4)^8$$

$$\int (3x-4)^8 \, dx = \frac{1}{27}(3x-4)^9 + c$$

Question 14   (2 marks)

Find $\int \dfrac{4 \sin\left(\frac{5x}{3}\right)}{7} \, dx$ **2**

$$\frac{d}{dx} \cos \frac{5x}{3} = -\frac{5}{3} \sin \frac{5x}{3}$$

$$= \frac{4}{7} \int \sin\left(\frac{5x}{3}\right) dx$$

$$= \frac{4}{7} \times \frac{3}{5} \int \frac{5}{3} \sin\left(\frac{5x}{3}\right) dx$$

$$= -\frac{12}{35} \cos\left(\frac{5x}{3}\right) + c$$

Question 15 on next page

Question 15   (3 marks)

A hiking group walked 8 km from their camp (C) on a bearing of 230° to point A.

They then walked 6 km on a bearing of 015° to point B.

The diagram below shows their path.

NOT TO SCALE
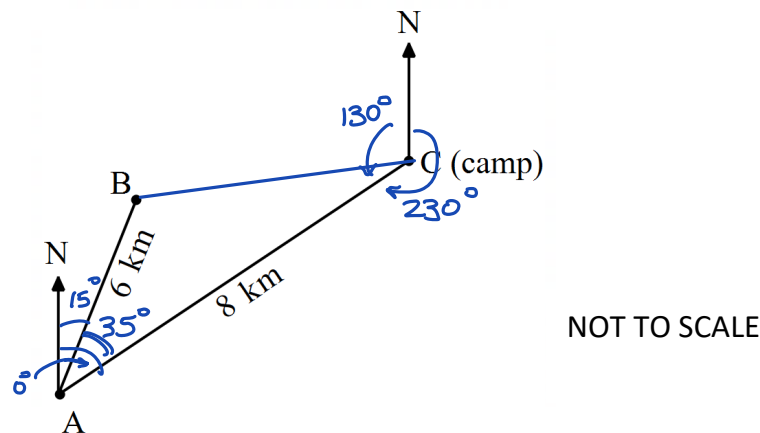
(a)   What is the size of $\angle CAB$?   **1**

$\angle NAB = 15°$ $\angle NAC = 50°$

$\therefore \angle CAB = 50 - 15$

$= 35°$

(b)   Calculate the distance from B to C, correct to 1 decimal place.   **2**

$BC^2 = 6^2 + 8^2 - 2(6)(8)\cos 35°$

$= 21.36$

$\therefore BC = 4.6 \text{ km}$

Question 16   (4 marks)

The curve C has the equation $y = f(x)$, $x \neq 0$. The point $P(2,1)$ lies on C.

(a) Find $f(x)$, given that $f'(x) = 3x^2 - 6 - \dfrac{8}{x^2}$. **2**

$$f(x) = \int (3x^2 - 6 - 8x^{-2})\, dx$$

$$= x^3 - 6x + 8x^{-1} + c$$

$$f(2) = (2)^3 - 6(2) + \frac{8}{2} + c = 1$$

$$\therefore c = 1$$

$$\therefore f(x) = x^3 - 6x + \frac{8}{x} + 1$$

(b) What is the equation of the tangent to C at the point P? **2**

$$m_{tangent} = 3(2)^2 - 6 - \frac{8}{(2)^2}$$

$$= 4$$

$$y - 1 = 4(x - 2)$$

$$= 4x - 8$$

$$\therefore y = 4x - 7 \qquad or \qquad 4x - y - 7 = 0$$

Question 17   (2 marks)
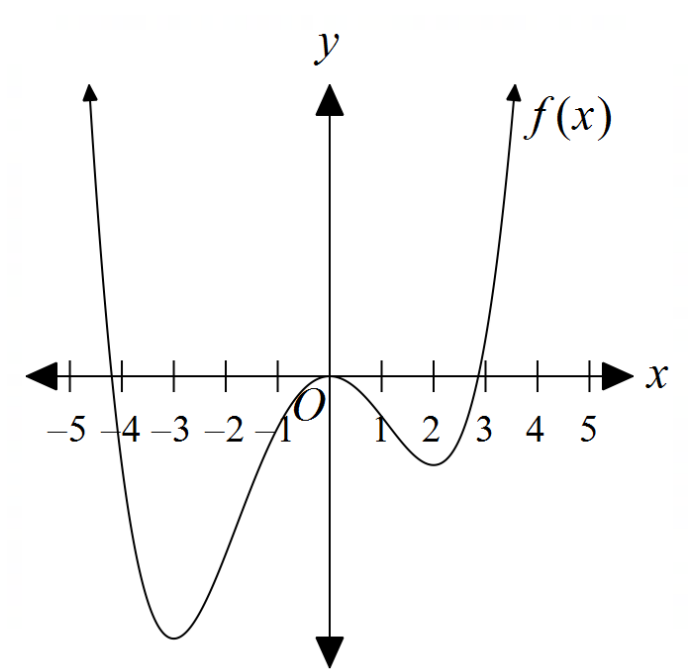
The diagram below shows the graph of $y = f(x)$.

In the space below, sketch the derivative function, $y = f'(x)$.   **2**
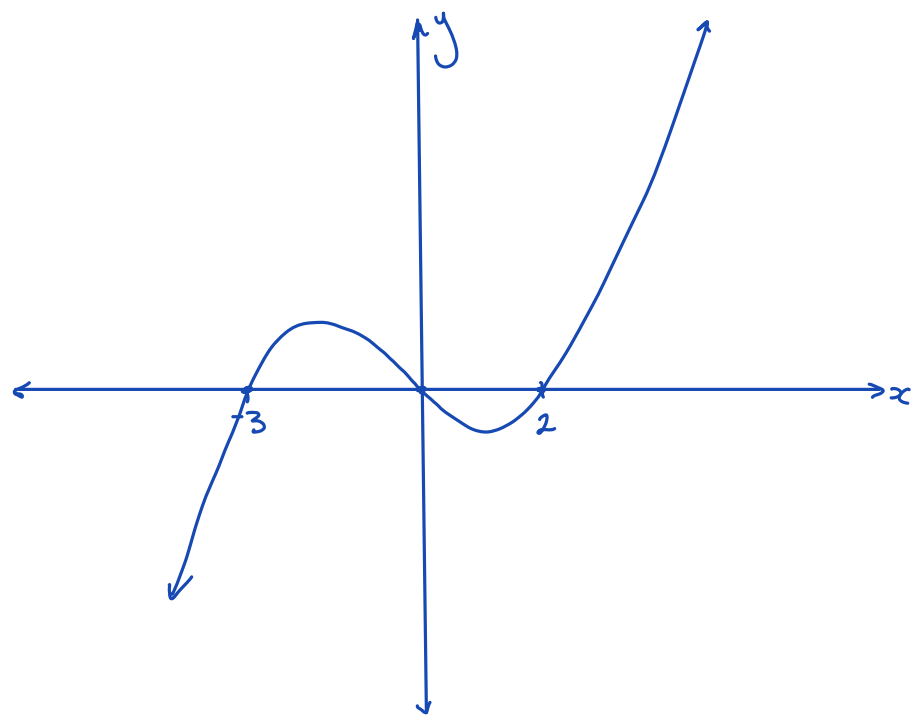
Question 18 on next page

Question 18   (2 marks)

Find $\int x^2 (x^3 + 5)^3 dx$ **2**

$$= \frac{1}{12} \int 12x^2 (x^3 + 5)^3 dx$$

$$= \frac{1}{12} (x^3 + 5)^4 + c$$

Question 19   (3 marks)

Consider the functions $f(x) = 2x^2 - 8x$ and $g(x) = x + 2$.

(a) Determine the composite function $f(g(x))$.   **2**

$$f(g(x)) = 2(x+2)^2 - 8(x+2)$$

$$= 2(x^2 + 4x + 4) - 8x - 16$$

$$= 2x^2 + 8x + 8 - 8x - 16$$

$$= 2x^2 - 8$$

$$= 2(x^2 - 4)$$

(b) Using interval notation, state the range of the composite function.   **1**

$$f(g(x)) = 2(x-2)(x+2)$$

Range: $[-8, \infty)$

Question 20   (2 marks)   **Marks**

The graph below shows $y = f(x)$.   **2**

On the same graph, sketch $y = 2 - f(x)$.
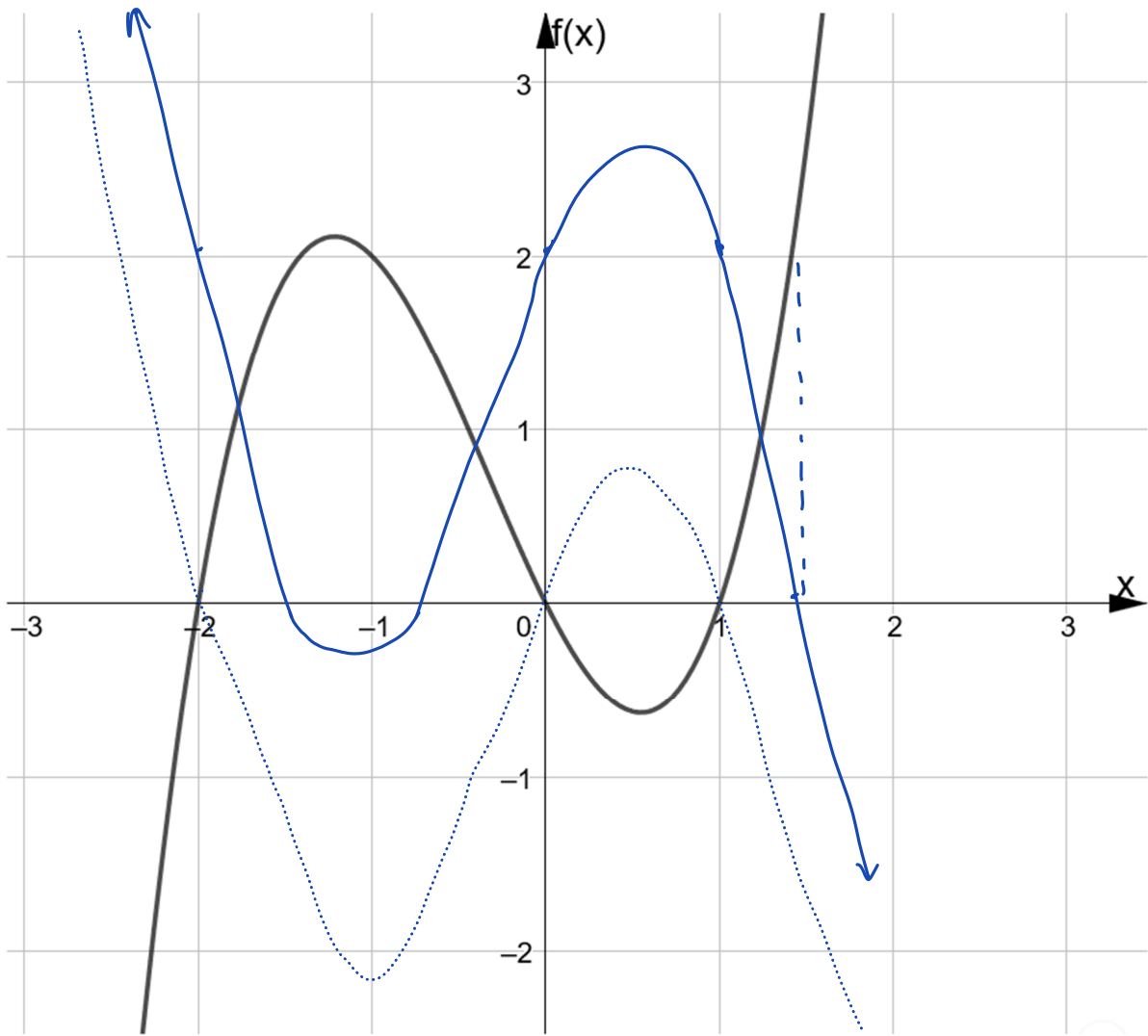
Question 21 on next page

Question 21   (2 marks)   **Marks**

Show that $\sin x + 1 + \cos x \cot x - \operatorname{cosec} x = 1$ **2**

$$\sin x + 1 + \cos x \cot x - \operatorname{cosec} x$$

$$= \sin x + 1 + \cos x \times \frac{\cos x}{\sin x} - \frac{1}{\sin x}$$

$$= \frac{\sin^2 x + \sin x + \cos^2 x - 1}{\sin x}$$

$$= \frac{1 + \sin x - 1}{\sin x}$$

$$= \frac{\sin x}{\sin x}$$

$$= 1 \qquad \text{as required}$$

Question 22   (2 marks)   **Marks**

Solve $2\cos(2x) = -\sqrt{3}$ for x where $0 \leq x \leq \pi$ **2**

$$\cos(2x) = -\frac{\sqrt{3}}{2}$$

S	A
T	C

$$2x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6} \ldots$$

$$x = \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{17\pi}{12} \ldots$$

Since $0 \leq x \leq \pi$, $x = \frac{5\pi}{12}$ or $\frac{7\pi}{12}$

Question 23 on next page

Question 23 (8 marks)

A function is given by $y = -x^3 + 9x^2 - 24x + 16$.

(a) Find any stationary points, determine their nature, and find any possible point/s of inflection.

$$\frac{dy}{dx} = -3x^2 + 18x - 24$$

$$= -3(x^2 - 6x + 8)$$

$$= -3(x-4)(x-2)$$

Stationary points when $\frac{dy}{dx} = 0$

$$x = 2 \quad \text{or} \quad x = 4$$

x	0	2	3	4	5
$\frac{dy}{dx}$	-24	0	3	0	-9
	↘	—	↗	—	↘

When $x = 2$, $y = -4$ When $x = 4$, $y = 0$

$\therefore (2, -4)$ is a local minimum stationary point

and $(4, 0)$ is a local maximum stationary point

$$\frac{d^2y}{dx^2} = -6x + 18$$

$$= -6(x-3)$$

Possible point of inflection when $\frac{d^2y}{dx^2} = 0$

$$\therefore x = 3, \quad y = -2$$

x	$\frac{5}{2}$	3	$\frac{7}{2}$	
$\frac{d^2y}{dx^2}$	3	0	-3	concavity changes

$\therefore$ Point of inflection at $(3, -2)$

Question 23 continued on next page

(b) Hence, or otherwise, sketch the curve labelling all important points.   **3**
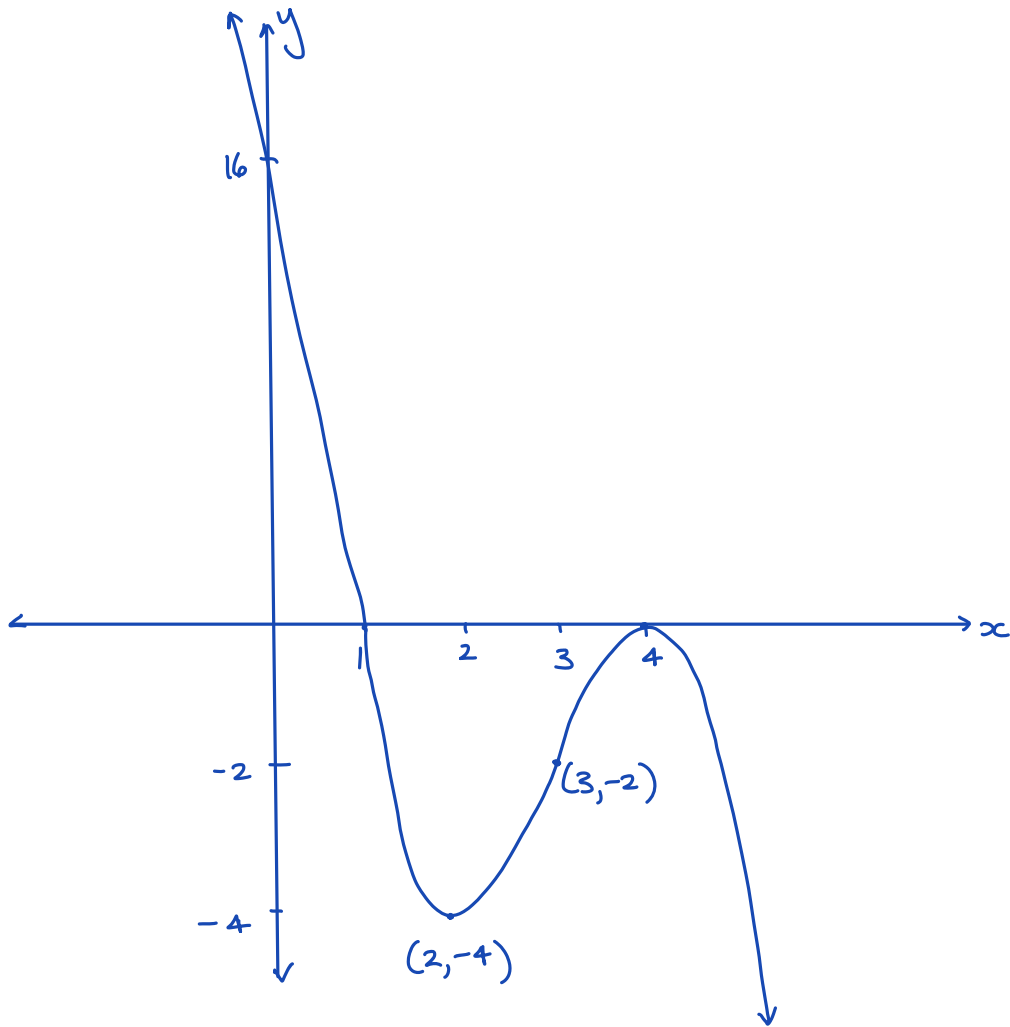
Question 24 on next page

Question 24   (3 marks)

(a) Complete the table of values for $y = \sqrt{1 - x^2}$.

    Answer to 3 decimal places where required.

$h = 0.125$

x	0	0.125	0.25	0.375	0.5
y	1	0.992	0.968	0.927	0.866

(b) Use the Trapezoidal rule with 4 subintervals, to estimate the value of

$$\int_0^{\frac{1}{2}} \sqrt{1 - x^2}\, dx$$

$$\int_0^{\frac{1}{2}} \sqrt{1-x^2}\, dx \doteqdot \frac{0.125}{2}\left[1 + 2(0.992 + 0.968 + 0.927) + 0.866\right]$$

$$\approx 0.4775$$

Question 25   (5 marks)

Lachlan works in a grocery store. He is making a stack of oranges against a sloping display panel.

The oranges are stacked in layers, as shown below. Each layer contains one orange less than the layer below it.
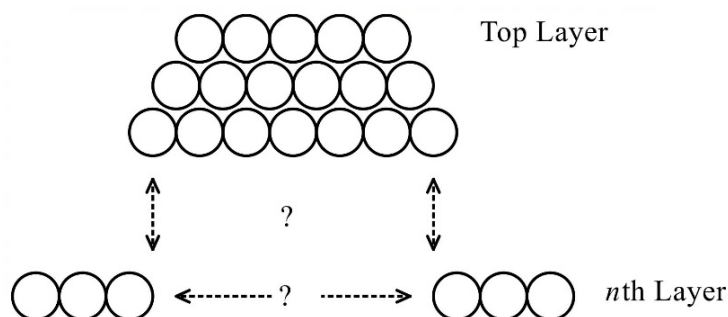
When he has finished, there are five oranges in the top layer, six in the next and so on.

There are n layers altogether.

<center>**Question 25 continued on next page**</center>

Question 25 continued

(a) Show that there are $\dfrac{1}{2}n(n+9)$ oranges in the stack. **2**

$$a = 5, \quad d = 1$$

$$S_n = \frac{n}{2}\left[2(5) + (n-1)\times 1\right]$$

$$= \frac{n}{2}\left[10 + n - 1\right]$$

$$= \frac{1}{2}n(n+9)$$

(b) If Lachlan has 300 oranges to create his display, how many full rows can he create, if the top row still contains five oranges? **3**

$$\text{If } S_n = 300,$$

$$\frac{1}{2}n(n+9) = 300$$

$$n^2 + 9n = 600$$

$$n^2 + 9n - 600 = 0$$

$$n = \frac{-9 \pm \sqrt{9^2 - 4(-600)(1)}}{2(1)}$$

$$\doteq 20.4 \quad \text{or} \quad -29.4$$

Since n is a positive integer, $n = 20$

Question 26   (6 marks)

From a packet of mixed seeds it was estimated that the probability of any seed planted yielding a red rose was 0.02.

(a) Calculate the probability that from any two seeds planted there will be: **1**

(i) two red roses

$$P(2 \text{ red roses}) = 0.02 \times 0.02$$
$$= 0.0004$$

(ii) at least one red rose   **2**

$$P(\text{at least one red}) = 1 - P(\text{no red})$$
$$= 1 - (0.98)^2$$
$$= 0.0396$$

(b) How many seeds must be planted for you to be at least 98% certain of obtaining at least one red rose?   **3**

$$P(\text{at least one red}) = 1 - (0.98)^n$$
$$0.98 = 1 - (0.98)^n$$
$$0.98^n = 0.02$$
$$n \ln 0.98 = \ln 0.02$$
$$n = \frac{\ln 0.02}{\ln 0.98}$$
$$= 193.638\ldots$$

∴ must plant 194 roses

Question 27 on next page

Question 27   (3 marks)   **Marks**

Differentiate $f(x) = 2x^2 + 5x$ from first principles.   **3**

$$f(x) = 2x^2 + 5x$$

$$f(x+h) = 2(x+h)^2 + 5(x+h)$$

$$= 2(x^2 + 2xh + h^2) + 5x + 5h$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 5x + 5h - 2x^2 - 5x}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 + 5h}{h}$$

$$= \lim_{h \to 0} (4x + 2h + 5)$$

$$= 4x + 5$$

Question 28 on next page

Question 28   (6 marks)

Two curves $y = 2e^{-x}$ and $y = e^x - 1$ intersect at point P.
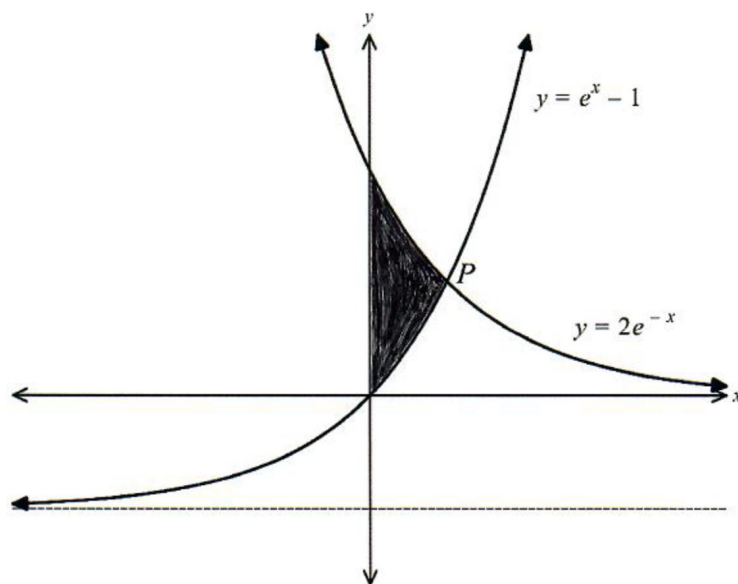
(a) Show that the coordinates of P are $(\ln 2, 1)$. **3**

Intersect when $2e^{-x} = e^x - 1$

$2 = e^{2x} - e^x$

$e^{2x} - e^x - 2 = 0$

$(e^x + 1)(e^x - 2) = 0$

$e^x = -1$ or $e^x = 2$

Since $e^x > 0$ for all real x

$\therefore e^x = 2$

$x = \ln 2$

When $x = \ln 2$, $y = e^{\ln 2} - 1 = 2 - 1 = 1$

$\therefore$ Intersect at $(\ln 2, 1)$

Question 28 continues on next page

Question 28 continued

(b) Calculate the shaded area. **3**

$$\text{Area} = \int_0^{\ln 2} \left(2e^{-x} - e^x + 1\right) dx$$

$$= \left[-2e^{-x} - e^x + x\right]_0^{\ln 2}$$

$$= \left(-2e^{-\ln 2} - e^{\ln 2} + \ln 2\right) - \left(-2e^0 - e^0 + 0\right)$$

$$= \left(-2\left(\tfrac{1}{2}\right) - 2 + \ln 2\right) - \left(-2 - 1\right)$$

$$= \ln 2 - 3 + 3$$

$$= \ln 2 \quad \text{units}^2$$

Question 29   (3 marks)   **Marks**

(a) Differentiate $\log_e(\cos x)$ with respect to x. **2**

$$\frac{d}{dx} \ln(\cos x) = \frac{-\sin x}{\cos x}$$

$$= -\tan x$$

(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \tan x \, dx$, correct to 2 decimal places. **2**

$$\int_0^{\frac{\pi}{4}} \tan x \, dx = -\left[\ln(\cos x)\right]_0^{\frac{\pi}{4}}$$

$$= -\left[\ln\left(\cos \tfrac{\pi}{4}\right) - \ln\left(\cos 0\right)\right]$$

$$= -\left[\ln\left(\tfrac{1}{\sqrt{2}}\right) - \ln(1)\right]$$

$$= -\left[\ln(1) - \ln(2)^{\frac{1}{2}} - \ln(1)\right]$$

$$= \tfrac{1}{2}\ln 2$$

$$\approx 0.35 \quad \text{to} \quad 2 \text{ decimal places}$$

A spinner has the numbers 1 to 5 placed on it. The probability distribution for the spinner is shown below.

x	1	2	3	4	5
$P(X = x)$	0.1	0.25	0.4	0.1	0.15

(a) Find $P(2 < X \leq 5)$ **1**

$P(2<X\leq5) = 0.4+0.1+0.15$

$= 0.65$

(b) Find $P(X \leq 3|X > 1)$ **1**

$P(X\leq3|X>1) = \dfrac{0.65}{1-0.1}$

$= 0.7\dot{2} \approx \dfrac{13}{18}$

(c) Find the expected value of X. **2**

$E(X) = 1(0.1)+2(0.25)+3(0.4)+4(0.1)+5(0.15)$

$= 2.95$

(d) Determine the variance of X. **2**

$Var(X) = E(X^2)-\mu^2$

$= [(0.1\times1^2)+(0.25\times2^2)+(0.4\times3^2)+(0.1\times4^2)+(0.15\times5^2)] - (2.95)^2$

$= 1.3475$

Question 31   (7 marks)

The bivariate data in the scatterplot below compares the recorded IQ Score (I) of students with their Mark on a Test (M) out of 100.
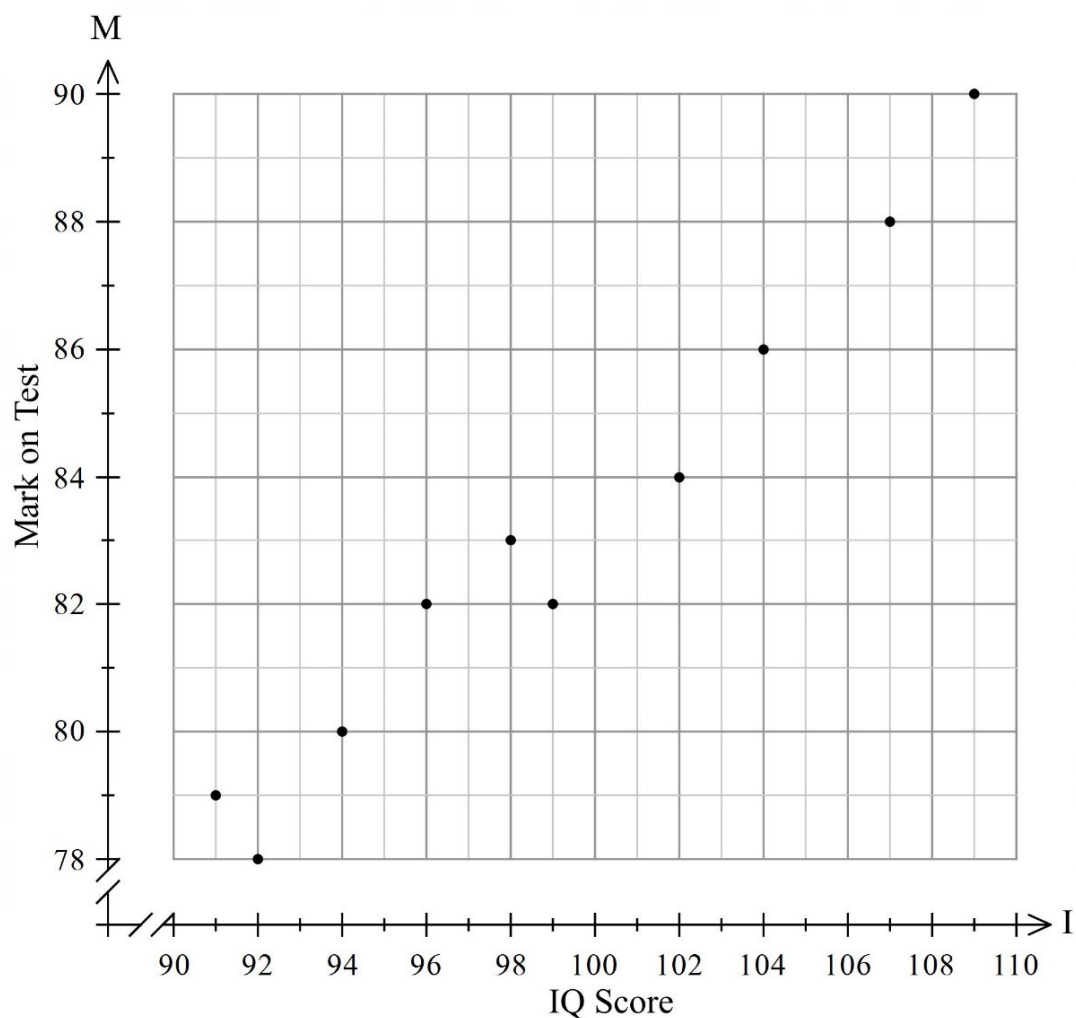
(a) Using your calculator, find (r) Pearson's correlation coefficient correct to 2
      decimal places and describe the direction and strength of correlation this
      data gives.

3

---

r = 0.98 to 2 decimal places

This is a strong, positive correlation

Question 31 continued on next page

(b) Using your calculator to find a line of best fit in the form $y = mx + c$. **2**
Round both m and c to two decimal places.

$$y = 22.18 + 0.62x$$

(c) Use your equation to estimate the Mark for a student who recorded an IQ **2**
score of 114.

Is this a valid estimate? Justify your answer.

$$y = 22.18 + 0.62(114)$$

$$= 92.3$$

Not a valid estimate as this value is
extrapolated outside the provided data.

Question 32 (2 marks) **Marks**

The graph below shows the velocity, v m/s, of a particle moving on a straight line,
for $0 \le t \le 9$.
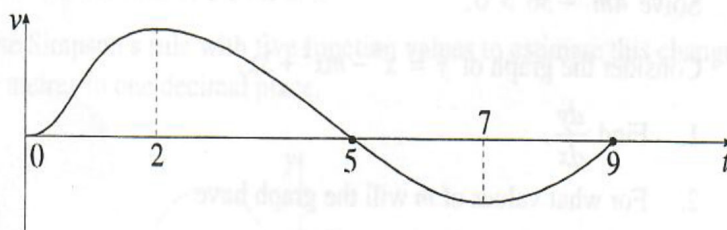
State all times, or interval of time, for which:

(a) the particle is stationary **1**

Stationary at $t = 0, 5$ and 9 seconds

(b) the acceleration is negative. **1**

Acceleration is negative when tangent is
negative ie. between 2 and 7 seconds

Question 33   (5 marks)

The water level in an estuary is cyclical, with a maximum depth of 5 metres, a minimum depth of 2 metres, and the cycle repeats every 12 hours.

The last high tide was at 2:00 am.

The function of the form $D = k\cos\frac{\pi}{6}(t + b) + c$ models the water depth, where D is the water depth (in metres), t is the hours since 12:00 am (midnight) and k, b and c are constants.
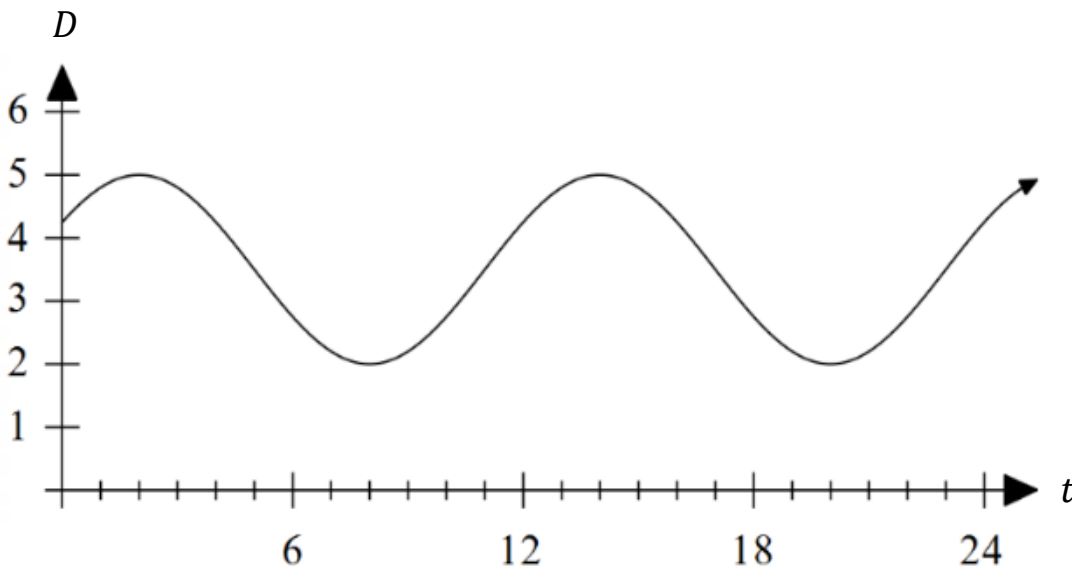
(a) Determine the values of k, b and c.   **3**

> Centre of motion is $\frac{5+2}{2} = 3.5$ metres
>
> ∴ $c = 3.5$
>
> $k = 1.5$
>
> $b = -2$
>
> Check: $1.5\cos\frac{\pi}{6}(2-2) + 3.5 = 5$ ✓

Question 33 continued on next page

Question 33 continued

(b) What is the rate of change of the water level at 12:00 pm? **2**

$$D = 1.5 \cos \frac{\pi}{6}(t-2) + 3.5$$

$$\frac{dD}{dt} = \frac{3}{2} \times \frac{-\pi}{6} \sin \frac{\pi}{6}(t-2)$$

$$= -\frac{\pi}{4} \sin \frac{\pi}{6}(t-2)$$

When $t = 12$,

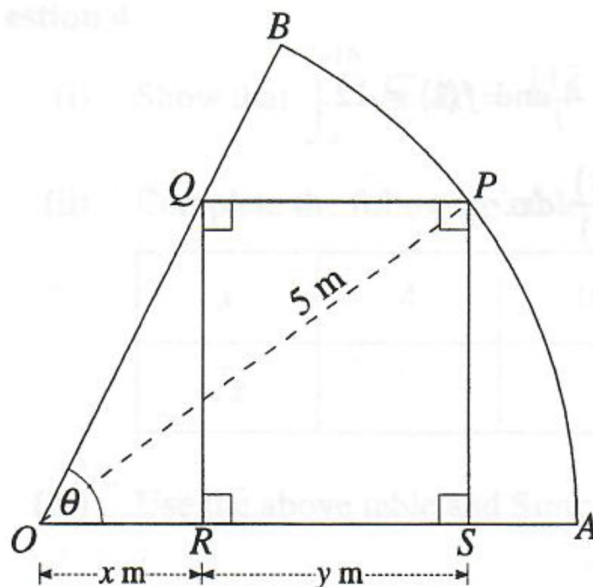$$\frac{dD}{dt} = -\frac{\pi}{4} \sin \frac{\pi}{6}(12-2)$$

$$= -\frac{\pi}{4} \sin \left(\frac{5\pi}{3}\right)$$

$$= -\frac{\pi}{4} \times -\frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}\pi}{8} \text{ metres/hour}$$

$$\approx 0.680 \text{ metres/hour}$$

Question 34 on next page

Question 34 (7 marks)

The diagram above shows a rectangle $PQRS$ drawn in a sector OAB of a circle with a radius of 5 metres.

$\angle AOB = \theta$ and $\tan\theta = 2$, $OR = x$ m and $RS = y$ m.

(a) Show that the perimeter, P m, of the rectangle is given by: **3**

$$P = 2x + 2\sqrt{25 - 4x^2}$$

$\tan\theta = \dfrac{QR}{OR}$

$\therefore 2 = \dfrac{QR}{x}$

$QR = 2x$

Considering $\triangle OPS$,

$5^2 = (x+y)^2 + (2x)^2$

$(x+y)^2 = 25 - 4x^2$

$x + y = \sqrt{25 - 4x^2}$

$y = \sqrt{25 - 4x^2} - x$

$\therefore$ Perimeter $= 2(2x) + 2(\sqrt{25-4x^2} - x)$

$= 2x + 2\sqrt{25-4x^2}$ as required

Question 34 continued on next page

Question 34 continued

(b) Find the maximum perimeter of the rectangle. **4**
Leave your answer in exact form.

$$P = 2x + 2(25 - 4x^2)^{\frac{1}{2}}$$

$$\frac{dP}{dx} = 2 + 2\left(\frac{1}{2}\right)(25 - 4x^2)^{-\frac{1}{2}} \times -8x$$

$$= 2 - \frac{8x}{\sqrt{25 - 4x^2}}$$

Max. perimeter when $\frac{dP}{dx} = 0$

$$\therefore\ 2 = \frac{8x}{\sqrt{25 - 4x^2}}$$

$$2\sqrt{25 - 4x^2} = 8x$$

$$4(25 - 4x^2) = 64x^2$$

$$100 - 16x^2 = 64x^2$$

$$80x^2 = 100$$

$$x^2 = \frac{5}{4}$$

$$x = \frac{\sqrt{5}}{2} \ \text{m} \ \approx 1.12\text{m}$$

x	1	$\frac{\sqrt{5}}{2}$	2
$\frac{dP}{dx}$	0.25	0	$-\frac{10}{3}$
	/	—	\

$$\therefore \text{max. perimeter} = 2\left(\frac{\sqrt{5}}{2}\right) + 2\sqrt{25 - 4\left(\frac{\sqrt{5}}{2}\right)^2}$$

$$= \sqrt{5} + 2\sqrt{25 - 4\left(\frac{5}{4}\right)}$$

$$= \sqrt{5} + 2\sqrt{20}$$

$$= \sqrt{5} + 4\sqrt{5}$$

$$= 5\sqrt{5} \ \text{metres}$$

End of Paper

Section II   Extra writing space

If you use this space, clearly indicate which question you are answering.

Section II   Extra writing space

If you use this space, clearly indicate which question you are answering.

Year 12 Mathematics Advanced Section I – Answer Sheet

Name: _____   **Class:** _____

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 =   (A) 2   (B) 6   (C) 8   (D) 9

A ◯   B ●   C ◯   D ◯

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ●   B ⊗   C ◯   D ◯

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct

A ●   B ⊗   C ◯   D ◯

1.   A ◯   B ◯   C ◯   D ◯

2.   A ◯   B ◯   C ◯   D ◯

3.   A ◯   B ◯   C ◯   D ◯

4.   A ◯   B ◯   C ◯   D ◯

5.   A ◯   B ◯   C ◯   D ◯

6.   A ◯   B ◯   C ◯   D ◯

7.   A ◯   B ◯   C ◯   D ◯

8.   A ◯   B ◯   C ◯   D ◯

9.   A ◯   B ◯   C ◯   D ◯

10. A ◯   B ◯   C ◯   D ◯